EISCAT_3D Project

WP5 Report on Synchronisation for interferometry

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Summary

It is shown that the phase of the visibility function (the observable in an interferometric measurement) depends directly and linearly on the time delay between the signals arriving at the two antennas of a baseline. Furthermore, one phase period maps to one fringe period, and each to one radio wave period. Thus, a useful condition on the accuracy of the phase — which determines the quality of the measured image — is that the timing system random variations (time jitter) should be a small fraction of the period of the radiation (the radar), $\delta << T_o$, T_o being the period of the radiation. In fact, the random time variation in this condition is the accumulated random time variation. Since the accumulated random time variation decreases in proportion to the inverse square root of the number of contributions when they are statistically independent, the actual individual contributions to the time jitter can be larger than the value determined by this condition. A practical condition can be that the total phase inaccuracy be about 10°, or 1/40th of a period, which gives 100 ps for a 250 MHz radar.

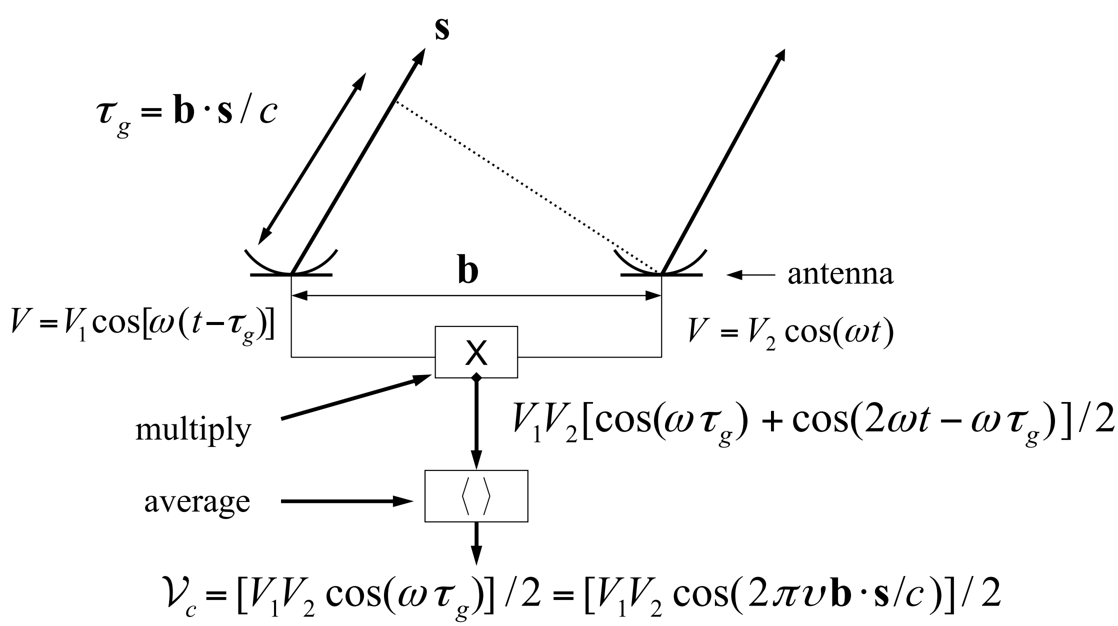
The calculation

The figure (based on a drawing by Rick Perley) on the next page shows the basic geometry of interferometry.

The time delay τ_g of the signals arriving at the two antennas determines the relationship between the observable — the cross-correlation between the signals (the voltages V_1, $V2$), called *visibility*, $\mathcal{V}$ — and the image — called *brightness* or *intensity*, $\mathcal{I}$. This relationship is one of a Fourier transformation between visibility and intensity. The time delay depends directly on the two complementary Fourier variables, which are determined by — or determine — the orientation of the antennas and the length of their baseline. We restrict momentarily the analysis to one dimension. The generalisation to two dimensions is straightforward. Thus:

$$\mathcal{V}(u) = \int_{-\infty}^{\infty} \mathcal{I}(l) e^{-i2\pi ul} dl \qquad (1)$$

Strictly, this equation is an approximation but it is valid in virtually all practical applications. We will postpone a description of the validity conditions for the time being.



The phase of the Fourier kernel in Eq. (1) is:

$$\phi = \omega_o \tau_g = 2\pi f_o \frac{\mathbf{b}\cdot\mathbf{s}}{c} = 2\pi \frac{b}{\lambda_o}\cos\alpha = 2\pi u l \tag{2}$$

where $\mathbf{b}$ is the baseline vector (in physical units, e.g. meters), $\mathbf{s}$ is a unit vector in the antenna pointing directions, ω_o, f_o and λ_o are the angular frequency, frequency and wavelength of the radiation (radar), and α is the direction cosine of the antenna pointing direction with respect to the baseline direction, or what is the same, the component of the unit vector $\mathbf{s}$ along the baseline. Thus, the visibility variable or domain u is the baseline measured in wavelengths, $u = b/\lambda_o$, and its inverse — the brightness or intensity variable or domain — is the direction cosine of the antenna pointing direction, $l = \cos\alpha$.

Eq. (2) can also be written as:

$$\phi = \omega_o \tau_g = 2\pi \frac{b}{\lambda_o}\cos\alpha = 2\pi \frac{\sin\theta}{\theta_o} \simeq 2\pi \frac{\theta}{\theta_o} \tag{3}$$

where θ is the complement of α, $\theta + \alpha = \pi/2$, that is, θ is the antenna zenith angle which is very small in virtually all practical applications. The fringe size

$$\theta_o = \frac{\lambda_o}{b} \tag{4}$$

is the angle variation of the antenna directions that produces a delay of one signal period, $T_o = 2\pi/\omega_o = 1/f_o$. The phase of the Fourier kernel, ϕ, changes by 2π when the zenith angle changes by one fringe size, which is the same as saying when the signal delay changes by one signal period. Therefore, we can adopt a practical accuracy requirement of the system time, also called time jitter:

$$\delta t << T_o \tag{5}$$

We suggest a tolerance of $\pm 10°$ or about 1/40th of a period. A frequency of 250 MHz has a period of 4 ns; therefore the relative accuracy of the system time is about ± 4 ns/40 = ± 100 ps.

Timing inaccuracies reduce sensitivity via a reduction of the visibility amplitude, thus decreasing the dynamic range and fidelity. In more clear language, they reduce the signal to noise ratio by decreasing the signal. It might be possible, under certain circumstances, to recover the lost amplitude. However, the SNR loss is irrecoverable.

It is remarkable that the required relative accuracy is independent of the baseline length. One would have thought that the timing accuracy would be determined by the longest baseline, which is the one that determines the ultimate resolution of the interferometer. It is good to confirm that democracy reigns over timing accuracy, at least in interferometry, if not elsewhere.

Conclusion

We have shown that the phase of the visibility function (the observable in an interferometric measurement) depends directly and linearly on the time delay between the signals arriving at the two antennas of a baseline. Furthermore, one phase period maps to one fringe period, and each to one radio wave period. Thus, a useful condition on the accuracy of the phase — which determines the quality of the measured image — is that the timing system random variations (time jitter) should be a small fraction of the period of the radiation (the radar), $\delta << T_o$. In fact, the random time variation in this condition is the accumulated random time variation. Since the accumulated random time variation decreases in proportion to the inverse square root of the number of contributions when they are statistically independent, the actual individual contributions to the time jitter can be larger than the value determined by this condition. For instance, in one antenna configuration under consideration, a module has 24×24=576 elements. Since in the beam forming process the signals from each element are added, the individual element time jitter can be larger than the target time jitter by a factor equal to $\sqrt{576} = 24$, provided that the contributions are statistically independent, which is a reasonable assumption.

A practical condition can be that the phase inaccuracy be about 10°, or 1/40th of a period, which gives 100 ps for a 250 MHz radar with period 4 ns. For the example above of a module with 576 elements, the individual time jitter contributions can be as large as 24×100 ps = 2.4 ns.

A important question is the phase inaccuracies contributed by baseline inaccuracies. Baseline lengths are difficult to measure with the necessary accuracy. The problem is solved in radio astronomy by frequent calibration of the phase using known sources. In a separate report we describe a procedure to calibrate the baselines of a radar interferometer using incoherent scattering signals from the ionosphere.